

RECEIVED: May 28, 2014

REVISED: August 7, 2014

ACCEPTED: August 17, 2014

PUBLISHED: September 1, 2014

# Texture and cofactor zeros of the neutrino mass matrix

Jiajun Liao,<sup>a</sup> D. Marfatia<sup>b</sup> and K. Whisnant<sup>a</sup>

<sup>a</sup>*Department of Physics and Astronomy, Iowa State University,  
Ames, IA 50011, U.S.A.*

<sup>b</sup>*Department of Physics and Astronomy, University of Hawaii at Manoa,  
Honolulu, HI 96822, U.S.A.*

*E-mail:* [jliao@iastate.edu](mailto:jliao@iastate.edu), [dmarf8@hawaii.edu](mailto:dmarf8@hawaii.edu), [whisnant@iastate.edu](mailto:whisnant@iastate.edu)

**ABSTRACT:** We study Majorana neutrino mass matrices that have two texture zeros, or two cofactor zeros, or one texture zero and one cofactor zero. The two texture/cofactor zero conditions give four constraints, which in conjunction with the five measured oscillation parameters completely determine the nine independent real parameters of the neutrino mass matrix. We also study the implications that future measurements of neutrinoless double beta decay and the Dirac CP phase will have on these cases.

**KEYWORDS:** Neutrino Physics, CP violation

**ARXIV EPRINT:** [1311.2639](https://arxiv.org/abs/1311.2639)

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Two texture zeros</b>	<b>2</b>
<b>3</b>	<b>Two cofactor zeros</b>	<b>7</b>
<b>4</b>	<b>One texture zero and one cofactor zero</b>	<b>8</b>
<b>5</b>	<b>Discussion</b>	<b>13</b>

---

## 1 Introduction

Neutrino phenomena are describable by the Majorana neutrino mass matrix,

$$M = V^* \text{diag}(m_1, m_2, m_3) V^\dagger, \quad (1.1)$$

where we work in the basis in which the charged lepton mass matrix is diagonal, the  $m_i$  are real and nonnegative,  $V = U \text{diag}(1, e^{i\phi_2/2}, e^{i\phi_3/2})$ , and

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}. \quad (1.2)$$

Here  $s_{ij}$  and  $c_{ij}$  stand for the sine and cosine of the mixing angles  $\theta_{ij}$ . We study the consequences of imposing two texture/cofactor zeros in the neutrino mass matrix. There are three classes of such ansatzes: two texture zeros (TT) [1–5], two cofactor zeros (CC) [6–10], and one texture zero and one cofactor zero (TC) [11]. Of the nine real parameters of  $M$ , five are fixed by measurements of the three mixing angles and two mass-squared differences; for a recent global three-neutrino fit see ref. [12]. The remaining four parameters, which we take to be the lightest mass, the Dirac phase, and the two Majorana phases, can then be determined from the four constraints that define the two texture/cofactor zeros. Consequently, the rate for neutrinoless double beta decay ( $0\nu\beta\beta$ ) which is given by the magnitude of the  $\nu_e - \nu_e$  element of the neutrino mass matrix,

$$|M_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 e^{-i\phi_2} s_{12}^2 c_{13}^2 + m_3 e^{-i\phi_3} s_{13}^2 e^{2i\delta}|, \quad (1.3)$$

is also determined.

In sections 2, 3 and 4, we use current experimental data to study the allowed parameter space for two texture zeros, two cofactor zeros, and one texture and one cofactor zero, respectively. We discuss and summarize our results in section 5.

## 2 Two texture zeros

The condition for a vanishing element  $M_{\alpha\beta} = M_{\alpha\beta}^* = 0$  is

$$m_1 U_{\alpha 1} U_{\beta 1} + m_2 e^{i\phi_2} U_{\alpha 2} U_{\beta 2} + m_3 e^{i\phi_3} U_{\alpha 3} U_{\beta 3} = 0. \quad (2.1)$$

Since there are two such constraints that depend linearly on the masses, the masses are related by

$$\frac{m_1}{c_1} = \frac{m_2 e^{i\phi_2}}{c_2} = \frac{m_3 e^{i\phi_3}}{c_3}, \quad (2.2)$$

where  $c_j$  are complex numbers that are quartic in the matrix elements of  $U$ . Then, with  $\delta m^2 = m_2^2 - m_1^2$  and  $\Delta m^2 = |m_3^2 - \frac{1}{2}(m_1^2 + m_2^2)|$ , we get two equations that relate  $m_1$  to the oscillation parameters and the Dirac phase  $\delta$ ,

$$m_1 = \sqrt{\frac{\delta m^2}{|c_2/c_1|^2 - 1}}, \quad (2.3)$$

$$m_1 = \sqrt{\frac{\frac{1}{2}\delta m^2 \pm \Delta m^2}{|c_3/c_1|^2 - 1}}, \quad (2.4)$$

where the plus and minus signs correspond to the normal hierarchy (NH) and the inverted hierarchy (IH), respectively. (For the NH the lightest mass is  $m_1$ , and for the IH the lightest mass is  $m_3 = \sqrt{m_1^2 + \frac{1}{2}\delta m^2 - \Delta m^2}$ .) For a fixed set of oscillation parameters each of these two equations give  $m_1$  as a function of  $\delta$ , and the intersections of the curves give the allowed values of  $m_1$  and  $\delta$ . We use the data from the latest global fit of ref. [12] to find the  $2\sigma$  allowed regions for the lightest mass and  $\delta$  that satisfy eqs. (2.3) and (2.4). Note that if we replace  $\delta$  by  $-\delta$ , the two constraints from the eqs. (2.3) and (2.4) will be the same since the magnitude of  $c_i$  does not depend on the sign of  $\delta$ , but because the latest global fit has a preference for negative values of  $\delta$  [12], the allowed regions for  $0 \leq \delta \leq 180^\circ$  are a little larger than for  $180^\circ \leq \delta \leq 360^\circ$ .

For two texture zeros in the mass matrix, there are  $\frac{6!}{2!4!} = 15$  different cases to consider. If two off-diagonal entries vanish, the mass matrices are block diagonal and have one neutrino decoupled from the others, which is inconsistent with the data. Therefore, we only need to consider 12 cases that can be divided into three categories:

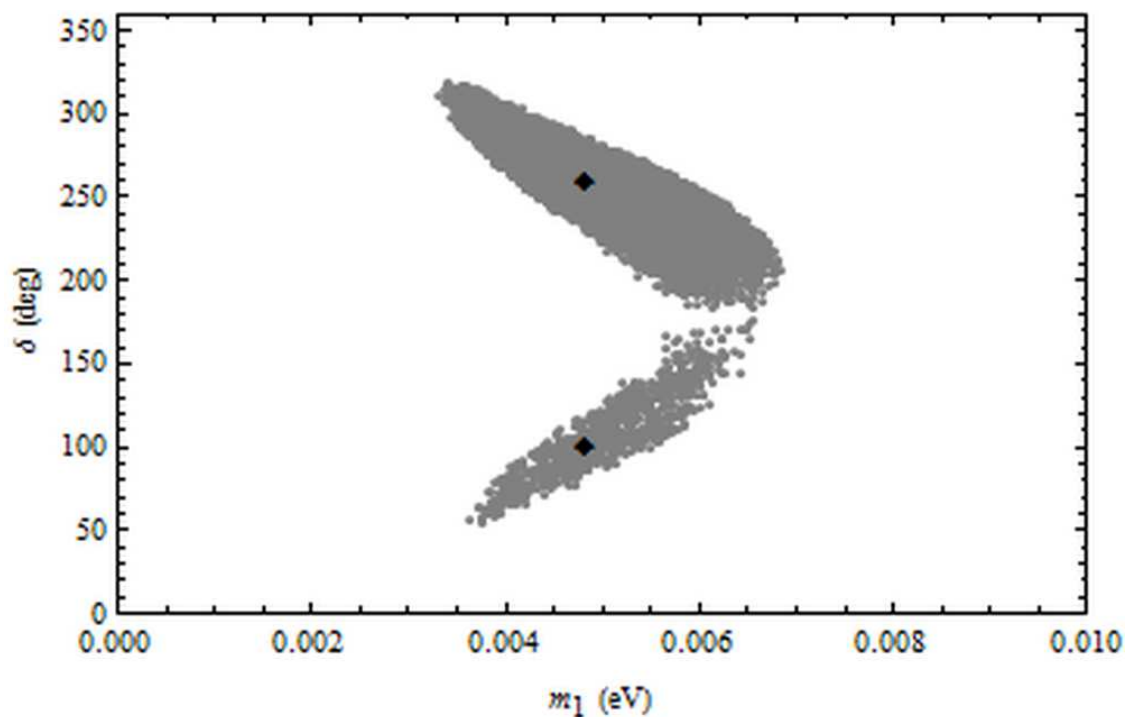
1. *One zero on diagonal, off-diagonal zero sharing column and row.* The six possibilities of this type,  $X_1, X_2, X_3, X_4, X_5$ , and  $X_6$ , are displayed in table 1. Using the unitarity of  $U$  and the fact that the cofactors of  $U_{ij}$  are equal to  $U_{ij}^*$ , e.g.,  $U_{e1}U_{\mu 2} - U_{e2}U_{\mu 1} = U_{\tau 3}^*$ , we obtain the simplified expressions for  $c_1, c_2$ , and  $c_3$  provided in table 1. From a numerical analysis, we find that at the  $2\sigma$  level, only  $X_1, X_2$  and  $X_5$  are allowed for the normal hierarchy and  $X_5$  and  $X_6$  are allowed for the inverted hierarchy. The allowed regions for  $X_1$  and  $X_2$  for the normal hierarchy are shown in figures 1 and 2. The allowed regions for  $X_5$  for the normal and inverted hierarchy are shown in figures 3 and 4, respectively. For the best-fit values of the measured oscillation parameters,  $X_2$  NH and  $X_5$  IH are not allowed, and the best-fit points for  $X_1$  NH and  $X_5$  NH are shown in figures 1 and 3 respectively. Both hierarchies for  $X_5$  have nearly maximal

Case	Structure	$c_1$	$c_2$	$c_3$
$X_1$	$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$U_{\tau 1}^* U_{e 2} U_{e 3}$	$U_{\tau 2}^* U_{e 3} U_{e 1}$	$U_{\tau 3}^* U_{e 1} U_{e 2}$
$X_2$	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$U_{\mu 1}^* U_{e 2} U_{e 3}$	$U_{\mu 2}^* U_{e 3} U_{e 1}$	$U_{\mu 3}^* U_{e 1} U_{e 2}$
$X_3$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$	$U_{e 1}^* U_{\tau 2} U_{\tau 3}$	$U_{e 2}^* U_{\tau 3} U_{\tau 1}$	$U_{e 3}^* U_{\tau 1} U_{\tau 2}$
$X_4$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix}$	$U_{e 1}^* U_{\mu 2} U_{\mu 3}$	$U_{e 2}^* U_{\mu 3} U_{\mu 1}$	$U_{e 3}^* U_{\mu 1} U_{\mu 2}$
$X_5$	$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$U_{\tau 1}^* U_{\mu 2} U_{\mu 3}$	$U_{\tau 2}^* U_{\mu 3} U_{\mu 1}$	$U_{\tau 3}^* U_{\mu 1} U_{\mu 2}$
$X_6$	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$U_{\mu 1}^* U_{\tau 2} U_{\tau 3}$	$U_{\mu 2}^* U_{\tau 3} U_{\tau 1}$	$U_{\mu 3}^* U_{\tau 1} U_{\tau 2}$

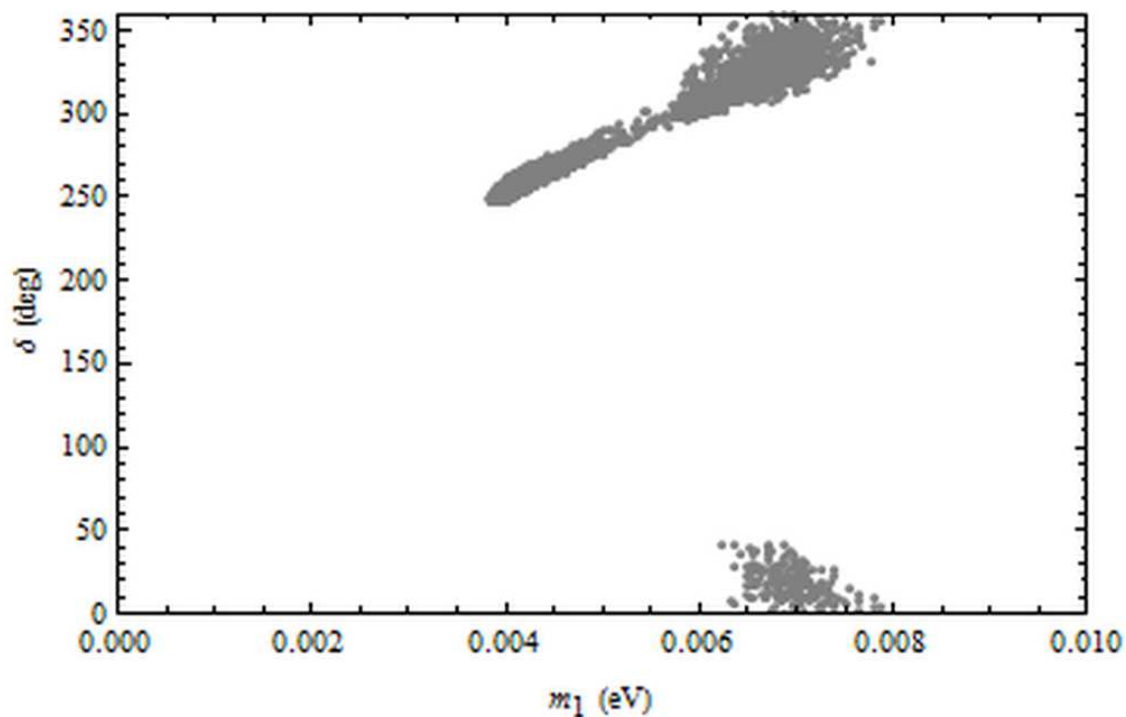
**Table 1.** The expressions for  $c_1$ ,  $c_2$  and  $c_3$  for Class X. The symbol  $\times$  denotes a nonzero matrix element.

CP violation, i.e.,  $\delta$  close to  $90^\circ$  or  $270^\circ$ , and a lower bound on the lightest mass of about 30 meV. For  $X_5$  NH and  $X_5$  IH, the upper bound on the lightest mass is about 290 meV and 250 meV, respectively. For comparison, the 95% C.L. limit from cosmology is  $\sum m_i < 660$  meV [13]. The allowed region for  $X_6$  IH is very similar to that for  $X_5$  IH.

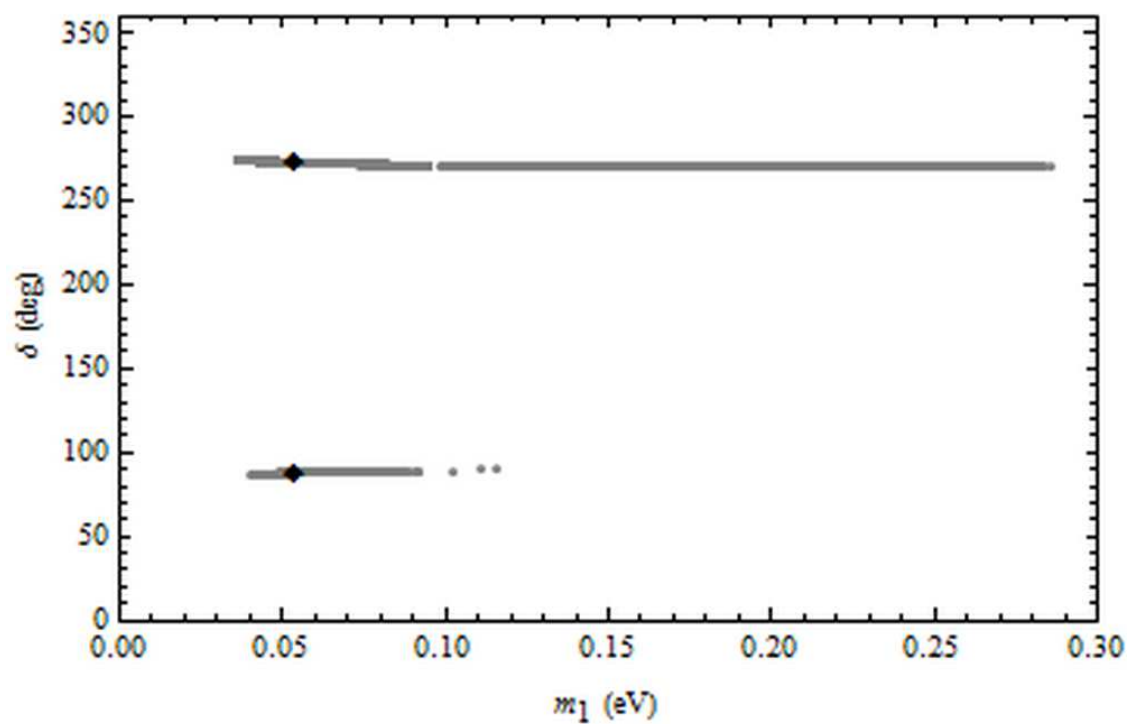
2. *One zero on diagonal, off-diagonal zero not sharing column and row.* The three possibilities of this type,  $Y_1$ ,  $Y_2$  and  $Y_3$ , and the corresponding  $c_i$ 's are displayed in table 2. At the  $2\sigma$  level,  $Y_1$  and  $Y_2$  are allowed for the inverted hierarchy, and their allowed regions are very similar to that for  $X_5$  IH;  $Y_1$  is also allowed for the normal hierarchy and the allowed region is very similar to that for  $X_5$  NH;  $Y_3$  is excluded at  $2\sigma$ . All the allowed cases have nearly maximal CP violation, and a lower bound on the lightest mass of about 30 meV, similar to  $X_5$  NH and  $X_5$  IH; see figures 3 and 4.
3. *Two zeros on diagonal.* The three possibilities of this type,  $Z_1$ ,  $Z_2$  and  $Z_3$ , and the corresponding  $c_i$ 's are listed in table 3. The numerical results show that only  $Z_1$  for



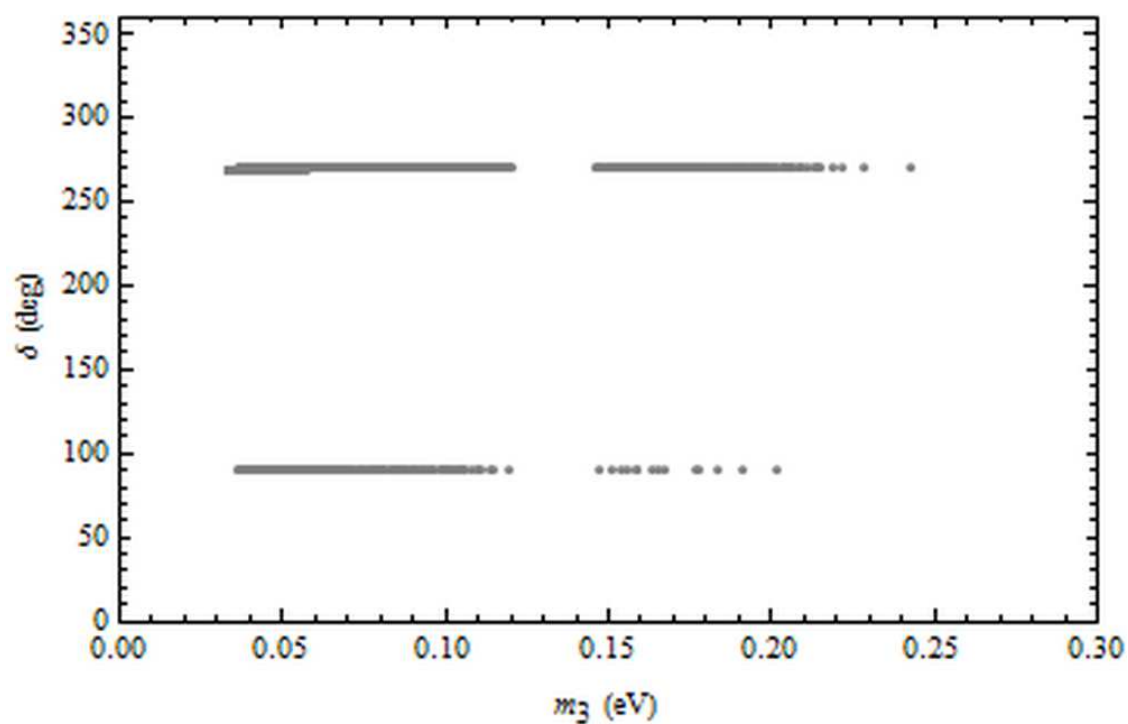
**Figure 1.** The  $2\sigma$  allowed regions in the  $(m_1, \delta)$  plane for the TT  $X_1$  case and the normal hierarchy. The black diamonds indicate  $m_1$  and  $\delta$  for the best-fit values of the five oscillation parameters.



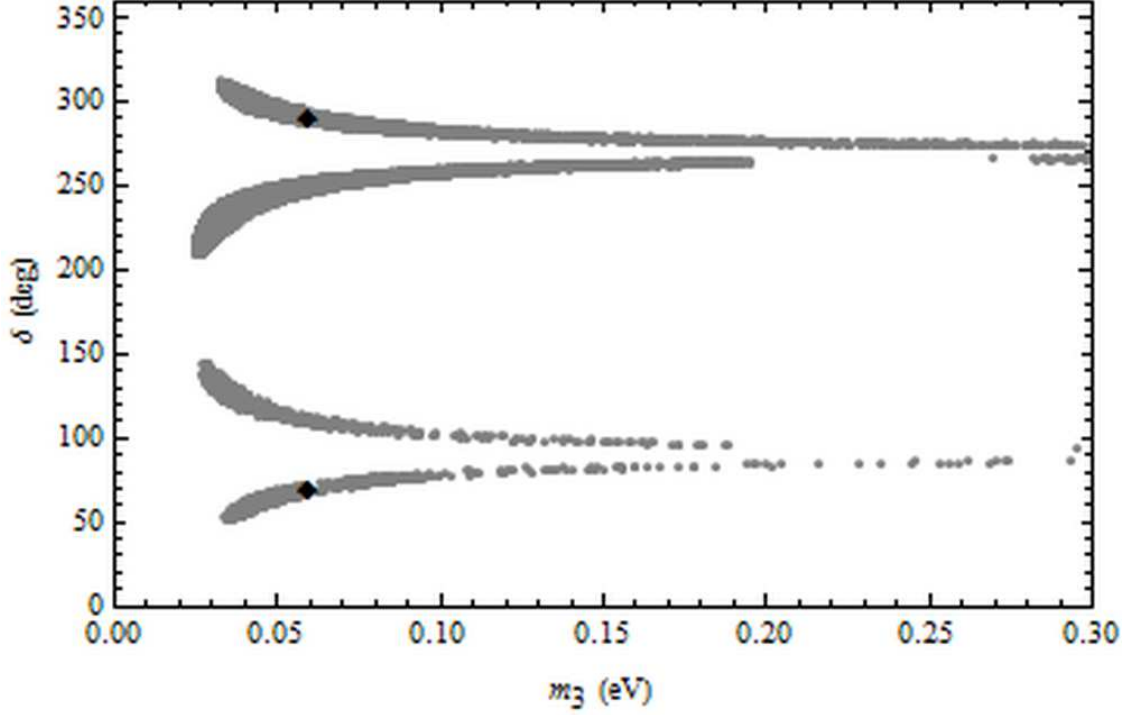
**Figure 2.** Same as figure 1, except for TT  $X_2$  and the normal hierarchy. This case is not allowed for the best-fit oscillation parameters.



**Figure 3.** Same as figure 1, except for TT  $X_5$  and the normal hierarchy.



**Figure 4.** Same as figure 1, except for TT  $X_5$  and the inverted hierarchy. This case is not allowed for the best-fit oscillation parameters.



**Figure 5.** Same as figure 1, except for TT  $Z_1$  and the inverted hierarchy.

Case	Structure	$c_1$	$c_2$	$c_3$
$Y_1$	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$U_{e1}^* U_{e2} U_{\mu 3} - U_{\tau 1}^* U_{\mu 2} U_{\tau 3}$	$U_{e1} U_{e2}^* U_{\mu 3} - U_{\mu 1} U_{\tau 2}^* U_{\tau 3}$	$U_{\mu 1} U_{e2} U_{e3}^* - U_{\tau 1} U_{\mu 2} U_{\tau 3}^*$
$Y_2$	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$U_{\mu 1}^* U_{\tau 2} U_{\mu 3} - U_{e1}^* U_{e2} U_{\tau 3}$	$U_{\tau 1} U_{\mu 2}^* U_{\mu 3} - U_{e1} U_{e2}^* U_{\tau 3}$	$U_{\mu 1} U_{\tau 2} U_{\mu 3}^* - U_{\tau 1} U_{e2} U_{e3}^*$
$Y_3$	$\begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix}$	$U_{\tau 1}^* U_{e2} U_{\tau 3} - U_{\mu 1}^* U_{\mu 2} U_{e3}$	$U_{e1} U_{\tau 2}^* U_{\tau 3} - U_{\mu 1} U_{\mu 2}^* U_{e3}$	$U_{\tau 1} U_{e2} U_{\tau 3}^* - U_{e1} U_{\mu 2} U_{\mu 3}^*$

**Table 2.** The expressions for  $c_1$ ,  $c_2$  and  $c_3$  for Class Y. The symbol  $\times$  denotes a nonzero matrix element.

the inverted hierarchy is allowed at the  $2\sigma$  level, and the allowed regions are shown in figure 5.  $Z_1$  for the normal hierarchy is excluded at  $2\sigma$  for  $m_1 < 0.3$  eV, which is consistent with the result of ref. [14].

Although the allowed regions for the seven acceptable textures of ref. [1] have been further restricted by the determination of  $\theta_{13}$ , all seven textures remain allowed. Further restrictions on the Dirac CP phase  $\delta$  [4, 5] can also be placed by the latest global fit [12] for each case.

Case	Structure	$c_1$	$c_2$	$c_3$
$Z_1$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$(U_{\mu 2} U_{\tau 3} + U_{\mu 3} U_{\tau 2}) U_{e 1}^*$	$(U_{\mu 3} U_{\tau 1} + U_{\mu 1} U_{\tau 3}) U_{e 2}^*$	$(U_{\mu 1} U_{\tau 2} + U_{\mu 2} U_{\tau 1}) U_{e 3}^*$
$Z_2$	$\begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$(U_{e 2} U_{\tau 3} + U_{e 3} U_{\tau 2}) U_{\mu 1}^*$	$(U_{e 3} U_{\tau 1} + U_{e 1} U_{\tau 3}) U_{\mu 2}^*$	$(U_{e 1} U_{\tau 2} + U_{e 2} U_{\tau 1}) U_{\mu 3}^*$
$Z_3$	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$(U_{e 2} U_{\mu 3} + U_{e 3} U_{\mu 2}) U_{\tau 1}^*$	$(U_{e 3} U_{\mu 1} + U_{e 1} U_{\mu 3}) U_{\tau 2}^*$	$(U_{e 1} U_{\mu 2} + U_{e 2} U_{\mu 1}) U_{\tau 3}^*$

**Table 3.** The expressions for  $c_1$ ,  $c_2$  and  $c_3$  for Class Z. The symbol  $\times$  denotes a nonzero matrix element.

### 3 Two cofactor zeros

In ref. [10] it was shown that for matrices with two zero cofactors, the lightest mass can vanish only if  $\theta_{13} = 0$ . Since  $\theta_{13}$  is nonzero at the  $7.7\sigma$  level [15], we assume there are no vanishing neutrino masses and the mass matrix is invertible. Since  $(M^{-1})_{\alpha\beta} = \frac{1}{\det M} C_{\beta\alpha}$  (where  $C_{\alpha\beta}$  is the  $(\alpha, \beta)$  cofactor of  $M$ ), and the Majorana neutrino mass matrix is symmetric,  $C_{\alpha\beta} = 0$  is equivalent to  $(M^{-1})_{\alpha\beta} = 0$ . Because  $M^{-1} = V \text{diag}(m_1^{-1}, m_2^{-1}, m_3^{-1}) V^T$ , the constraint is

$$m_1^{-1} U_{\alpha 1} U_{\beta 1} + m_2^{-1} e^{i\phi_2} U_{\alpha 2} U_{\beta 2} + m_3^{-1} e^{i\phi_3} U_{\alpha 3} U_{\beta 3} = 0. \quad (3.1)$$

The above equation is the same as eq. (2.1), except that the  $m_i$ 's are replaced by their inverses. Hence, we can follow a procedure similar to that for the TT case to find the favored values of the lightest mass and  $\delta$ . Since the cofactor matrix is also diagonalized by the mixing matrix  $V$ , it cannot be block diagonal, and only 12 different patterns need to be considered. It is possible to employ the notation for the TT case if the locations of the two zeros are the same in the cofactor matrix as in the mass matrix. Then all the  $c_i$ 's are identical, and the only difference from the TT case is that eqs. (2.3) and (2.4) are replaced by

$$m_1 = \sqrt{\frac{\delta m^2}{|c_1/c_2|^2 - 1}}, \quad (3.2)$$

$$m_1 = \sqrt{\frac{\frac{1}{2}\delta m^2 \pm \Delta m^2}{|c_1/c_3|^2 - 1}}. \quad (3.3)$$

As for the TT case, there are three categories:

1. *One zero on diagonal, off-diagonal zero sharing column and row.* An interesting fact is that the two cofactor zero cases in this class yield the same allowed regions as for the two texture zero cases in the same class [6–10]; the correspondence is listed in



Two cofactor zeros	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$
Two texture zeros	$X_3$	$X_4$	$X_1$	$X_2$	$X_6$	$X_5$

**Table 4.** The correspondence between the two cofactor zero cases and two texture zero cases for Class X.

table 4. The reason for this is that the two cofactor zero conditions in this category imply either two texture zeros, or three cofactor zeros in a row or column. The latter possibility which gives a vanishing mass is excluded since  $\theta_{13} \neq 0$ . From table 4, we readily find the cases that are allowed at  $2\sigma$ :  $X_3$ ,  $X_4$  and  $X_6$  for the normal hierarchy, and  $X_5$  and  $X_6$  for the inverted hierarchy. The allowed regions in the  $m_1(m_3)$ - $\delta$  plane are the same as those for the corresponding cases in the TT ansatz.

2. *One zero on diagonal, off-diagonal zero not sharing column and row.* There are three possibilities of this type:  $Y_1$ ,  $Y_2$  and  $Y_3$ ; see table 2. At the  $2\sigma$  level,  $Y_1$  and  $Y_2$  are allowed for the inverted hierarchy, and their allowed regions are very similar to that for TT  $X_5$  IH;  $Y_2$  is also allowed for the normal hierarchy and the allowed region is very similar to that for TT  $X_5$  NH;  $Y_3$  is excluded at  $2\sigma$ . All the allowed cases have nearly maximal CP violation, and a lower bound on the lightest mass of about 30 meV, similar to TT  $X_5$  NH and and TT  $X_5$  IH.
3. *Two zeros on diagonal.* There are three possibilities of this type:  $Z_1$ ,  $Z_2$  and  $Z_3$ ; see table 3. We find numerically that  $Z_1$  is allowed at  $2\sigma$  for the normal hierarchy only. The other cases are excluded at  $2\sigma$ . The allowed regions for  $Z_1$  for the normal hierarchy are shown in figure 6.

## 4 One texture zero and one cofactor zero

There are 36 possibilities with one texture zero and one cofactor zero, of which 21 are equivalent to a TT case [11]. So we only need to study the remaining 15 cases listed in table 5. The two constraints  $M_{\alpha\beta} = 0$  and  $C_{\alpha'\beta'} = 0$  can be written as

$$m_1 A_1 + m_2 e^{-i\phi_2} A_2 + m_3 e^{-i\phi_3} A_3 = 0, \quad (4.1)$$

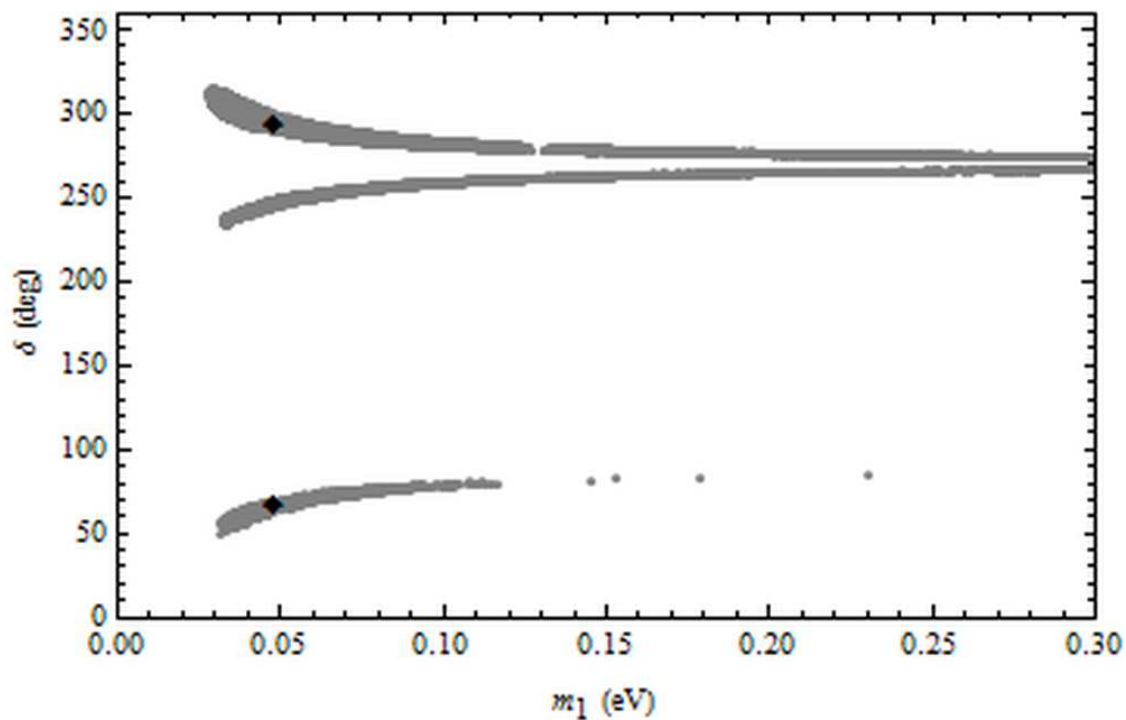
and

$$m_1^{-1} B_1 + m_2^{-1} e^{i\phi_2} B_2 + m_3^{-1} e^{i\phi_3} B_3 = 0, \quad (4.2)$$

where  $A_i = U_{\alpha i}^* U_{\beta i}^*$ , and  $B_i = U_{\alpha' i} U_{\beta' i}$  for  $i = 1, 2, 3$ . Solving these two equations, we get

$$\frac{m_3}{m_1} e^{-i\phi_3} = \frac{1}{2A_3 B_1} (A_2 B_2 - A_1 B_1 - A_3 B_3 \pm \sqrt{\Lambda}), \quad (4.3)$$

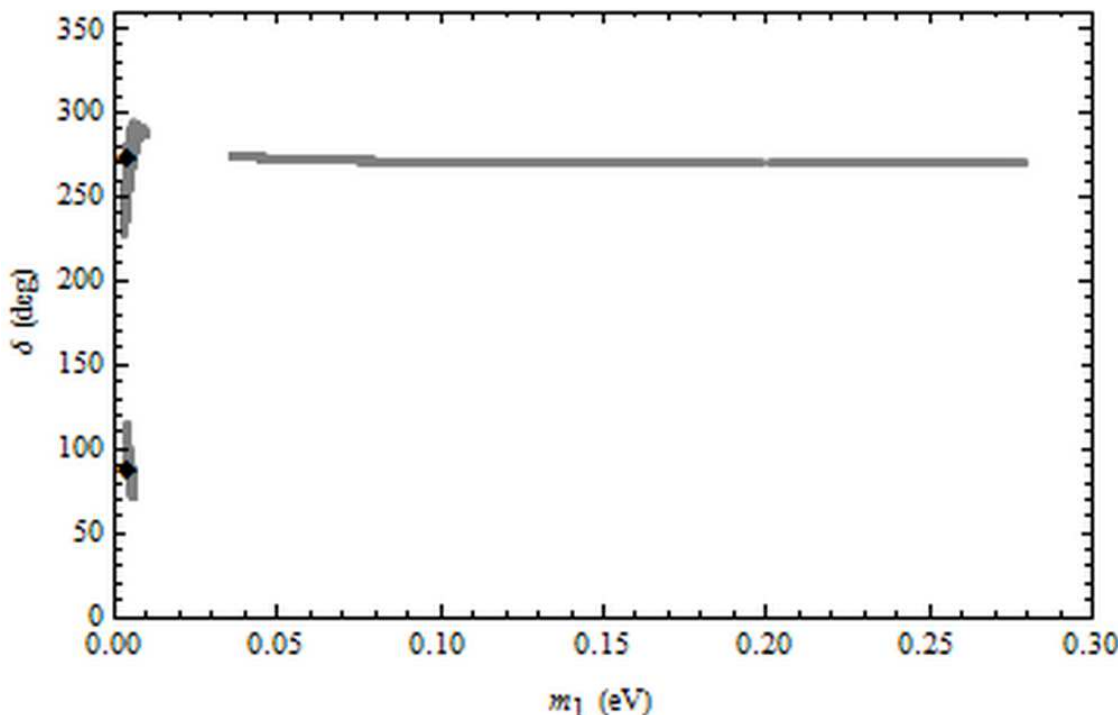
$$\frac{m_2}{m_1} e^{-i\phi_2} = \frac{1}{2A_2 B_1} (A_3 B_3 - A_1 B_1 - A_2 B_2 \mp \sqrt{\Lambda}), \quad (4.4)$$



**Figure 6.** Same as figure 1, except for CC  $Z_1$  and the normal hierarchy.

Case	Conditions
1A	$M_{ee} = 0, C_{ee} = 0$
1B	$M_{ee} = 0, C_{e\mu} = 0$
1C	$M_{ee} = 0, C_{e\tau} = 0$
2A	$M_{e\mu} = 0, C_{ee} = 0$
2D	$M_{e\mu} = 0, C_{\mu\mu} = 0$
3A	$M_{e\tau} = 0, C_{ee} = 0$
3F	$M_{e\tau} = 0, C_{\tau\tau} = 0$
4B	$M_{\mu\mu} = 0, C_{e\mu} = 0$
4D	$M_{\mu\mu} = 0, C_{\mu\mu} = 0$
4E	$M_{\mu\mu} = 0, C_{\mu\tau} = 0$
5D	$M_{\mu\tau} = 0, C_{\mu\mu} = 0$
5F	$M_{\mu\tau} = 0, C_{\tau\tau} = 0$
6C	$M_{\tau\tau} = 0, C_{e\tau} = 0$
6E	$M_{\tau\tau} = 0, C_{\mu\tau} = 0$
6F	$M_{\tau\tau} = 0, C_{\tau\tau} = 0$

**Table 5.** The 15 cases with one texture zero and one cofactor zero that are not reducible to a TT case.



**Figure 7.** Same as figure 1, except for TC 2D and the normal hierarchy.

where  $\Lambda = A_1^2 B_1^2 + A_2^2 B_2^2 + A_3^2 B_3^2 - 2(A_1 A_2 B_1 B_2 + A_1 A_3 B_1 B_3 + A_2 A_3 B_2 B_3)$ . Taking the absolute values of the above equations, we can find the two mass ratios,  $\sigma = m_2/m_1$  and  $\rho = m_3/m_1$ . Then,

$$\begin{aligned} m_1 &= \sqrt{\frac{\delta m^2}{\sigma^2 - 1}}, \\ m_1 &= \sqrt{\frac{\frac{1}{2}\delta m^2 \pm \Delta m^2}{\rho^2 - 1}}, \end{aligned} \quad (4.5)$$

A numerical study shows that at  $2\sigma$ , only 2D, 3F and 4B are allowed for the normal hierarchy, and only 2A, 2D, 3A, 3F, 4B and 6C are allowed for the inverted hierarchy. The allowed regions for 2D and 3F for the normal hierarchy are shown in figures 7 and 8, and the allowed regions for 2A, 4B and 6C for the inverted hierarchy are shown in figures 9, 10 and 11, respectively. The allowed region for 3A IH is very similar to that for 2A IH. The allowed region for 4B NH is very similar to that for TT  $X_5$  NH, and the allowed regions for 2D IH and 3F IH are very similar to that for TT  $X_5$  IH. They have nearly maximal CP violation, and a lower bound on the lightest mass of about 30 meV. For 3F NH and 6C IH there are four best-fit points since there are four solutions to the one texture zero and one cofactor zero conditions.

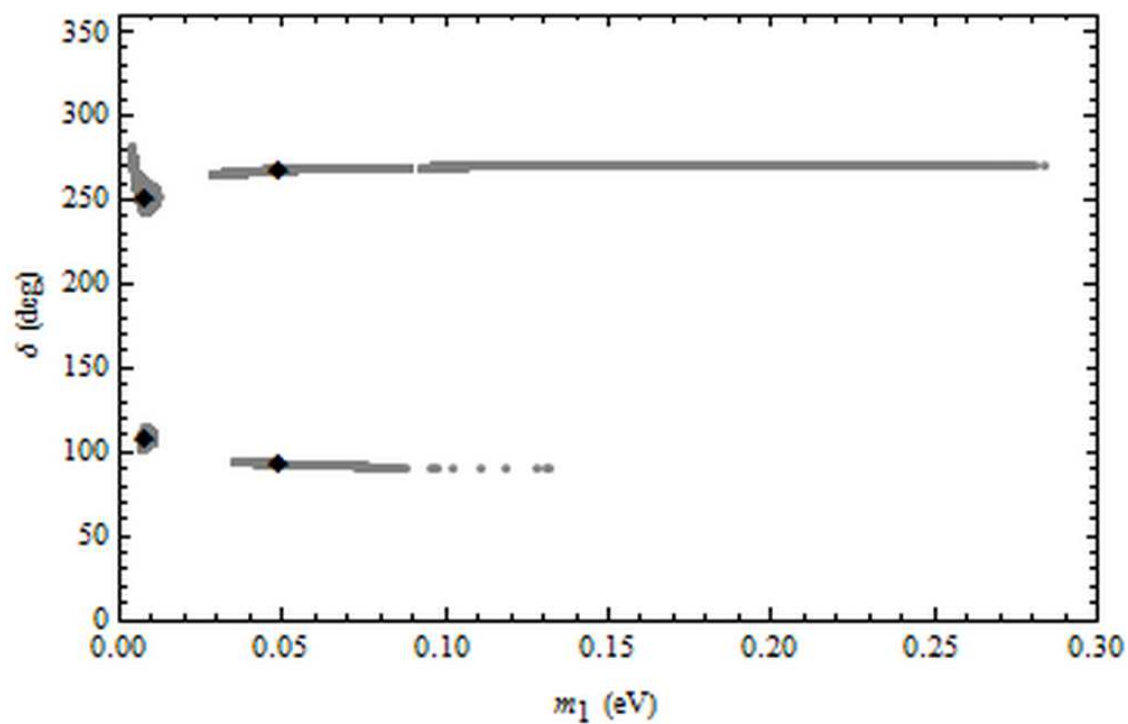


Figure 8. Same as figure 1, except for TC  $3F$  and the normal hierarchy.

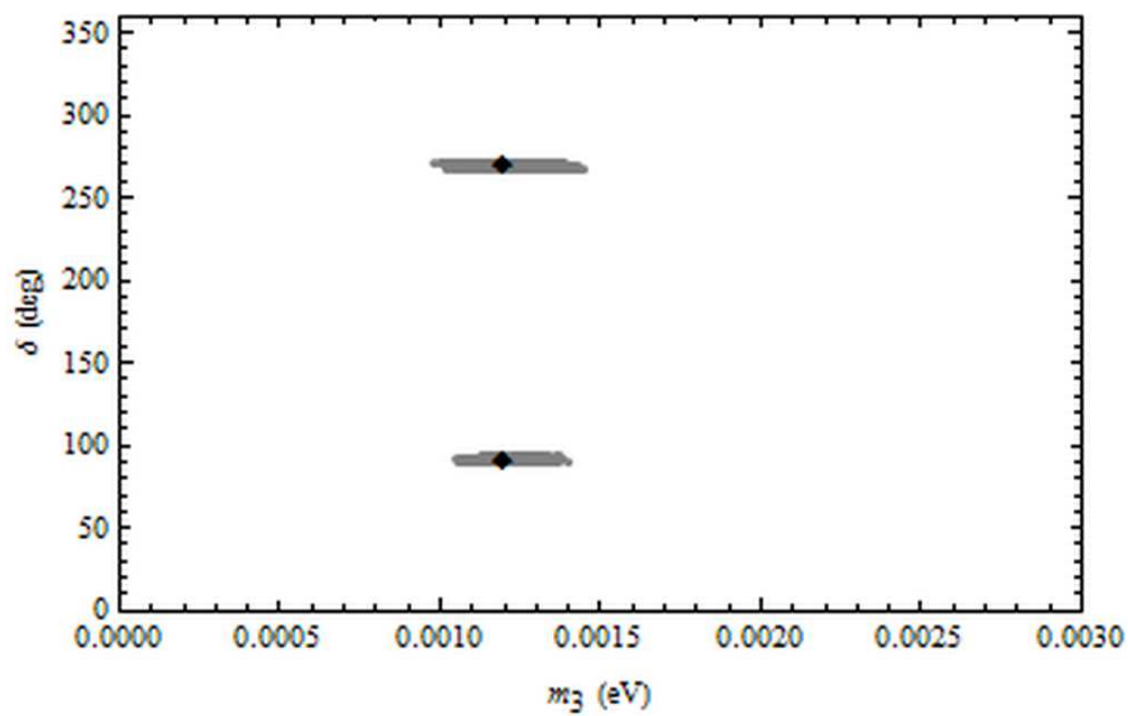
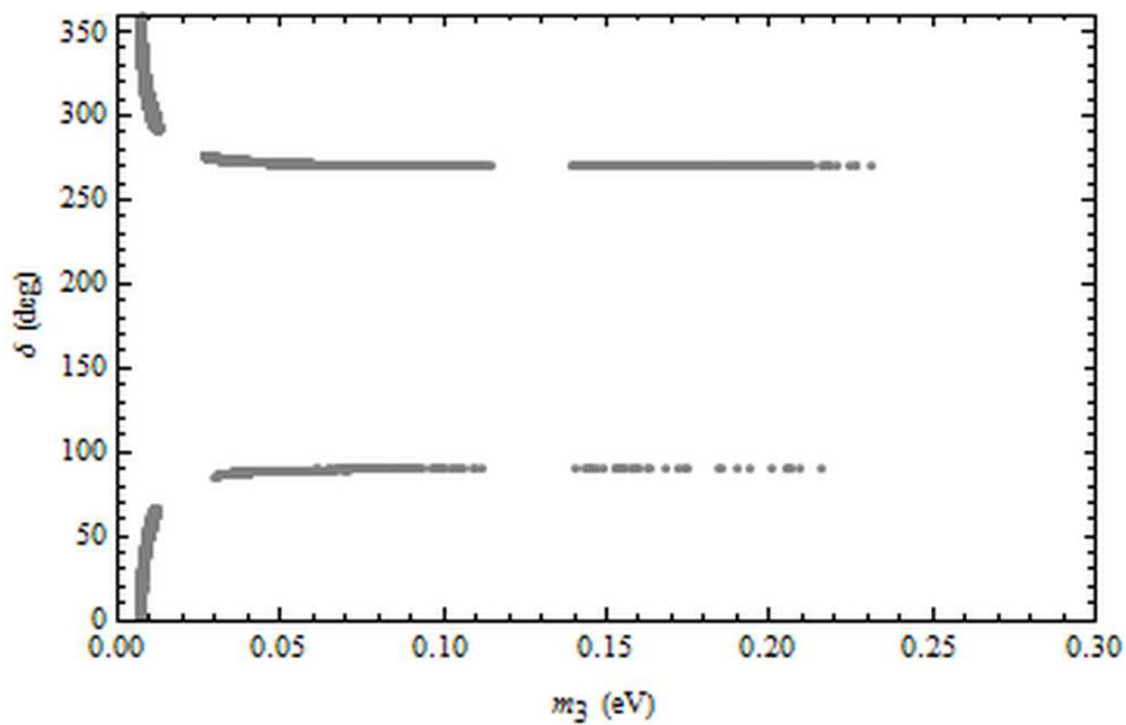
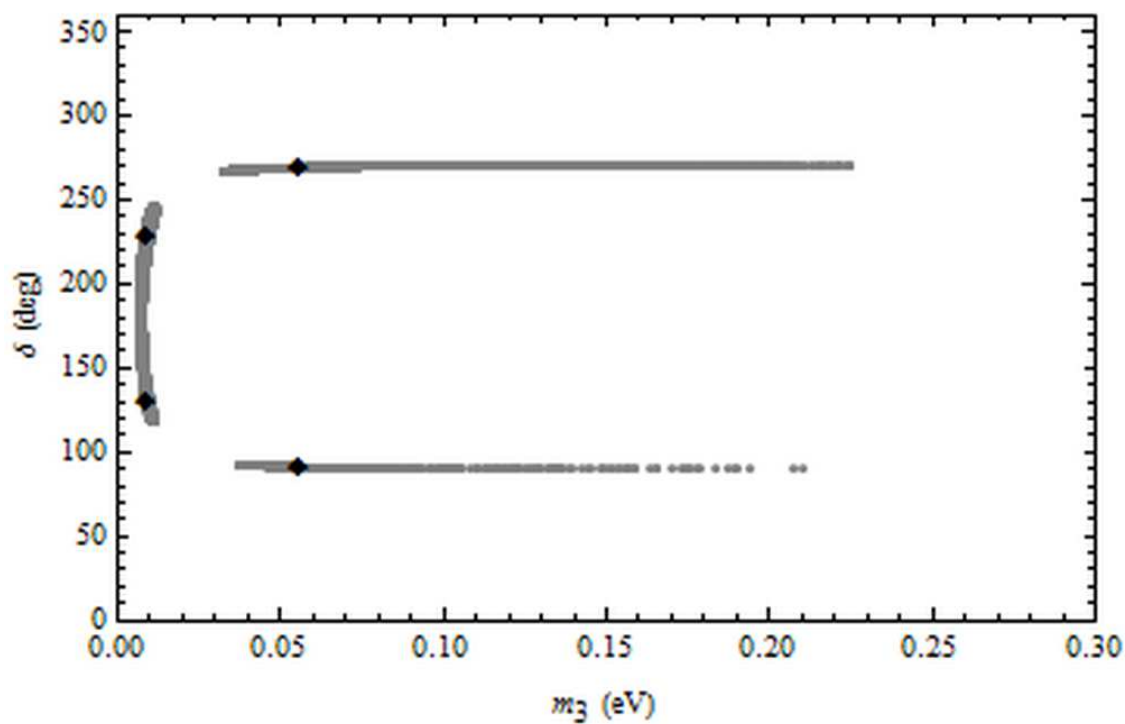


Figure 9. Same as figure 1, except for TC  $2A$  and the inverted hierarchy.



**Figure 10.** Same as figure 1, except for TC 4B and the inverted hierarchy. This case is not allowed for the best-fit oscillation parameters.



**Figure 11.** Same as figure 1, except for TC 6C and the inverted hierarchy. There are four best-fit points since there are four solutions to the TC conditions.

## 5 Discussion

There are 7 cases that are allowed at the  $2\sigma$  level for the two texture zero ansatz, 7 cases that are allowed for the two cofactor zero ansatz, and 6 cases that are allowed for the one texture and one cofactor zero ansatz. Seven cases allow both hierarchies, so there are a total of 27 possible two-zero cases allowed at  $2\sigma$ . However, there are many similarities among the allowed regions for these cases. In ref. [16] we noted that any case with a homogeneous relationship among elements of  $M$  with one mass hierarchy yields predictions for the oscillation parameters and phases similar to those given by a case with the same homogeneous relationship among cofactors of  $M$  with the opposite mass hierarchy. The only exceptions are when the lightest mass is small, of order 20 meV or less, or when the allowed ranges of the oscillation parameters differ significantly for the two mass hierarchies. The latter situation occurs for  $\theta_{23}$ , which is constrained at the  $2\sigma$  level to be less than about  $45.3^\circ$  for the NH but can have larger values for the IH.

A texture or cofactor zero is the simplest homogeneous relationship; therefore, CC cases can be dual to TT cases (and, of course, vice versa), and some TC cases can be dual to other such cases.<sup>1</sup> We can identify 8 cases where allowed regions are similar due to the dual-case argument, 6 cases where a case is allowed at  $2\sigma$  but its dual case is not because the lightest mass is small, 5 cases where an IH case is allowed but its dual case is disfavored because  $\theta_{23}$  must be larger than  $45.3^\circ$ . A complete listing of dual case relationships is given in table 6.

We note that the allowed regions for the CC  $Z_1$  NH case (figure 6) are similar to the allowed regions for its dual case, TT  $Z_1$  IH (figure 5). The region for  $90^\circ \leq \delta \leq 180^\circ$  in figure 5 does not appear in figure 6 because  $\theta_{23}$  has values that are larger than  $45.3^\circ$  for the inverted hierarchy for  $90^\circ \leq \delta \leq 180^\circ$ , while such values of  $\theta_{23}$  are disfavored for the normal hierarchy.

We can also use the effective Majorana mass for the neutrinoless double beta decay to differentiate two-zero cases. In table 7, we list the minimum and maximum values of  $|M_{ee}|$  at the  $2\sigma$  level for each case. Note that for TT  $X_1$  and  $X_2$ , and CC  $X_3$  and  $X_4$ ,  $|M_{ee}|$  is identically zero, and therefore they are not listed in the table. We also omit the cases of CC  $X_5$  IH,  $X_6$  NH and  $X_6$  IH because they give the same phenomenology as the corresponding cases in the TT class, as given in table 4.

We find that there are four different types of cases phenomenologically:

1. *Cases that allow only a small value for the lightest mass, less than 10 meV.* This includes 6 cases: TT  $X_1$  NH, TT  $X_2$  NH, CC  $X_3$  NH, CC  $X_4$  NH, TC 2A IH and TC 3A IH (figures 1, 2 and 9, respectively). The value of  $|M_{ee}|$  is either zero (for the TT cases) or close to 50 meV (for the TC cases).
2. *Cases that restrict  $\delta$  to be very close to  $90^\circ$  or  $270^\circ$ .* This group consists of 5 cases with NH (TT  $X_5$ , CC  $X_6$ , TT  $Y_1$ , CC  $Y_2$ , and TC 4B) and 10 cases with IH (TT  $X_5$ , TT  $X_6$ , CC  $X_5$ , CC  $X_6$ , TT  $Y_1$ , CC  $Y_1$ , TT  $Y_2$ , CC  $Y_2$ , TC 2D, and TC

<sup>1</sup>In principle, the TC cases 1A, 2D, and 6F could be self-dual, which means they would have similar allowed regions for the NH and IH, but these are not allowed at  $2\sigma$ .

Case	Hierarchy	Dual case allowed?
TT $X_1$	NH	No, $m_1$ small
TT $X_2$	NH	No, $m_1$ small
TT $X_5$	NH	Yes, CC $X_5$ IH
TT $X_5$	IH	Maybe, CC $X_5$ NH, if the $\theta_{23}$ restriction were absent
TT $X_6$	IH	Yes, CC $X_6$ NH
TT $Y_1$	NH	Yes, CC $Y_1$ IH
TT $Y_1$	IH	Maybe, CC $Y_1$ NH, if the $\theta_{23}$ restriction were absent
TT $Y_2$	IH	Yes, CC $Y_2$ NH
TT $Z_1$	IH	Yes, CC $Z_1$ NH (for $\delta \in [0, 90^\circ] \cup [180^\circ, 360^\circ]$ )
CC $X_3$	NH	No, $m_1$ small
CC $X_4$	NH	No, $m_1$ small
CC $X_6$	IH	Maybe, TT $X_6$ NH, if the $\theta_{23}$ restriction were absent
CC $Y_2$	IH	Maybe, TT $Y_2$ NH, if the $\theta_{23}$ restriction were absent
TC 2A	IH	No, $m_3$ small
TC 2D	NH	Yes, TC 4B IH (except for low $m_1$ and $m_3$ and if the $\theta_{23}$ restriction were absent)
TC 2D	IH	Yes, TC 4B NH
TC 3A	IH	No, $m_3$ small
TC 3F	NH	Yes, TC 6C IH (except for low $m_1$ and $m_3$ )
TC 3F	IH	Maybe, TC 6C NH, if the $\theta_{23}$ restriction were absent

**Table 6.** A listing of which allowed cases have dual cases that are also allowed, and which do not. The “Maybe” designation is for situations in which the dual case has a NH and  $\theta_{23} > 45.3^\circ$ ; the global analysis of ref. [12] suggests that for a NH,  $\theta_{23} < 45.3^\circ$  at  $2\sigma$ . “Maybe” indicates that the exclusion of the dual case on this basis is not robust.

3F), all of which have a minimum value for the lightest mass of about 30 meV. In all NH cases of this type, the maximum value for  $m_1$  is about 290 meV and  $35 \text{ meV} \lesssim |M_{ee}| \lesssim 290 \text{ meV}$ ; in all IH cases, the maximum value for  $m_3$  is about 250 meV and  $55 \text{ meV} \lesssim |M_{ee}| \lesssim 250 \text{ meV}$ . Therefore it will be very difficult to distinguish these cases from each other.

3. *Cases in which maximal CP violation is approached for larger values of the lightest mass.* This group includes TT  $Z_1$  IH and CC  $Z_1$  NH (figures 5 and 6). In these cases a wide range of  $\delta$  is possible, although maximal CP violation is not allowed.
4. *Cases that are a mixture of types 1 and 2.* The TC cases 2D NH, 3F NH, 4B IH, and 6C IH allow values of the lightest mass less than 10 meV, and also have nearly maximal CP violation when the lightest mass is above about 30 meV (figures 7, 8, 10, and 11).

Case	Hierarchy	Minimum	Maximum
TT $X_5$	NH	37	286
TT $X_5$	IH	58	247
TT $X_6$	IH	62	215
TT $Y_1$	NH	34	276
TT $Y_1$	IH	57	230
TT $Y_2$	IH	60	226
TT $Z_1$	IH	24	>1000
CC $Y_1$	IH	59	231
CC $Y_2$	NH	34	275
CC $Y_2$	IH	56	227
CC $Z_1$	NH	15	>1000
TC $2A$	IH	45	49
TC $2D$	NH	3	279
TC $2D$	IH	60	227
TC $3A$	IH	45	49
TC $3F$	NH	3	281
TC $3F$	IH	57	217
TC $4B$	NH	35	281
TC $4B$	IH	16	232
TC $6C$	IH	15	229

**Table 7.** The minimum and maximum values of  $|M_{ee}|$  (in meV) at  $2\sigma$ . CC  $X_5$  and CC  $X_6$  are not shown since they are equivalent to TT  $X_6$  and TT  $X_5$ , respectively.  $|M_{ee}|$  is identically zero for TT  $X_1$ , TT  $X_2$ , CC  $X_3$  and CC  $X_4$ .

Due to the large number of cases and their overlapping predictions, it is currently not possible to uniquely determine any given case. The latest experimental result from EXO-200 [17] sets an upper limit on the effective mass  $|M_{ee}|$  of less than 140 – 380 meV at 90% C.L. However, with future sensitivities to  $|M_{ee}|$  of about 20 meV [18], and a precision measurement of  $\delta$  in future long baseline oscillation experiments, we might be able to distinguish between these cases. Here we run a test on the survivability of two-zero cases by applying an upper limit on  $|M_{ee}|$  and assuming specific values for  $\delta$  with the  $3\sigma$  resolution attainable with a 350 kt-yr exposure at the Long-Baseline Neutrino Experiment [19]. The results in table 8 are qualitative without specific confidence levels ascribable.

## Acknowledgments

KW thanks the Center for Theoretical Underground Physics and Related Areas (CETUP 2013) in South Dakota for its hospitality and for partial support while this work was in



Case	$ M_{ee}  < 20 \text{ meV}$				$ M_{ee}  < 50 \text{ meV}$				$ M_{ee}  < 100 \text{ meV}$			
	0	90°	180°	270°	0	90°	180°	270°	0	90°	180°	270°
TT $X_1$ NH	×	✓	✓	✓	×	✓	✓	✓	×	✓	✓	✓
TT $X_2$ NH	✓	✓	×	✓	✓	✓	×	✓	✓	✓	×	✓
TT $X_5$ NH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
TT $X_5$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TT $X_6$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TT $Y_1$ NH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
TT $Y_1$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TT $Y_2$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TT $Z_1$ IH	×	×	×	×	×	✓	✓	✓	×	✓	✓	✓
CC $Y_1$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
CC $Y_2$ NH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
CC $Y_2$ IH	×	×	×	×	×	×	×	×	×	✓	×	✓
CC $Z_1$ NH	×	✓	×	✓	×	✓	×	✓	×	✓	×	✓
TC 2A IH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
TC 2D NH	×	✓	×	✓	×	✓	×	✓	×	✓	×	✓
TC 2D IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TC 3A IH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
TC 3F NH	×	✓	×	✓	×	✓	×	✓	×	✓	×	✓
TC 3F IH	×	×	×	×	×	×	×	×	×	✓	×	✓
TC 4B NH	×	×	×	×	×	✓	×	✓	×	✓	×	✓
TC 4B IH	✓	✓	×	✓	✓	✓	×	✓	✓	✓	×	✓
TC 6C IH	×	✓	✓	✓	×	✓	✓	✓	×	✓	✓	✓

**Table 8.** The two-zero cases that survive (indicated by a tick mark) an upper limit on  $|M_{ee}|$  and a measurement of  $\delta$  (as in the second row) with the  $3\sigma$  resolution attainable by the Long-Baseline Neutrino Experiment with 350 kt-yr of data [19]. The CC Class X is not shown since it is equivalent to the TT Class X.

progress. This research was supported by the U.S. Department of Energy under Grant Nos. DE-FG02-01ER41155, DE-FG02-13ER42024 and DE-SC0010504.

**Open Access.** This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

## References

- [1] P.H. Frampton, S.L. Glashow and D. Marfatia, *Zeroes of the neutrino mass matrix*, *Phys. Lett. B* **536** (2002) 79 [[hep-ph/0201008](#)] [[INSPIRE](#)].
- [2] Z.-z. Xing, *Texture zeros and Majorana phases of the neutrino mass matrix*, *Phys. Lett. B* **530** (2002) 159 [[hep-ph/0201151](#)] [[INSPIRE](#)].

- [3] Z.-z. Xing, *A Full determination of the neutrino mass spectrum from two zero textures of the neutrino mass matrix*, *Phys. Lett. B* **539** (2002) 85 [[hep-ph/0205032](#)] [[INSPIRE](#)].
- [4] H. Fritzsch, Z.-z. Xing and S. Zhou, *Two-zero textures of the Majorana neutrino mass matrix and current experimental tests*, *JHEP* **09** (2011) 083 [[arXiv:1108.4534](#)] [[INSPIRE](#)].
- [5] P.O. Ludl, S. Morisi and E. Peinado, *The reactor mixing angle and CP-violation with two texture zeros in the light of T2K*, *Nucl. Phys. B* **857** (2012) 411 [[arXiv:1109.3393](#)] [[INSPIRE](#)].
- [6] L. Lavoura, *Zeros of the inverted neutrino mass matrix*, *Phys. Lett. B* **609** (2005) 317 [[hep-ph/0411232](#)] [[INSPIRE](#)].
- [7] S. Verma, *Non-zero  $\theta_{13}$  and CP-violation in inverse neutrino mass matrix*, *Nucl. Phys. B* **854** (2012) 340 [[arXiv:1109.4228](#)] [[INSPIRE](#)].
- [8] S. Dev, S. Gupta, R.R. Gautam and L. Singh, *Near maximal atmospheric mixing in neutrino mass matrices with two vanishing minors*, *Phys. Lett. B* **706** (2011) 168 [[arXiv:1111.1300](#)] [[INSPIRE](#)].
- [9] T. Araki, J. Heeck and J. Kubo, *Vanishing minors in the neutrino mass matrix from abelian gauge symmetries*, *JHEP* **07** (2012) 083 [[arXiv:1203.4951](#)] [[INSPIRE](#)].
- [10] E.I. Lashin and N. Chamoun, *Zero minors of the neutrino mass matrix*, *Phys. Rev. D* **78** (2008) 073002 [[arXiv:0708.2423](#)] [[INSPIRE](#)].
- [11] S. Dev, S. Verma, S. Gupta and R.R. Gautam, *Neutrino mass matrices with a texture zero and a vanishing minor*, *Phys. Rev. D* **81** (2010) 053010 [[arXiv:1003.1006](#)] [[INSPIRE](#)].
- [12] F. Capozzi et al., *Status of three-neutrino oscillation parameters, circa 2013*, *Phys. Rev. D* **89** (2014) 093018 [[arXiv:1312.2878](#)] [[INSPIRE](#)].
- [13] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, *Astron. Astrophys.* (2014) [[arXiv:1303.5076](#)] [[INSPIRE](#)].
- [14] D. Meloni, A. Meroni and E. Peinado, *Two-zero Majorana textures in the light of the Planck results*, *Phys. Rev. D* **89** (2014) 053009 [[arXiv:1401.3207](#)] [[INSPIRE](#)].
- [15] DAYA BAY collaboration, F.P. An et al., *Improved measurement of electron antineutrino disappearance at Daya Bay*, *Chin. Phys. C* **37** (2013) 011001 [[arXiv:1210.6327](#)] [[INSPIRE](#)].
- [16] J. Liao, D. Marfatia and K. Whisnant, *Dual models of the neutrino mass spectrum*, *Phys. Rev. D* **89** (2014) 013009 [[arXiv:1308.1368](#)] [[INSPIRE](#)].
- [17] EXO collaboration, M. Auger et al., *Search for neutrinoless double-beta decay in  $^{136}\text{Xe}$  with EXO-200*, *Phys. Rev. Lett.* **109** (2012) 032505 [[arXiv:1205.5608](#)] [[INSPIRE](#)].
- [18] W. Rodejohann, *Neutrinoless double beta decay and neutrino physics*, *J. Phys. G* **39** (2012) 124008 [[arXiv:1206.2560](#)] [[INSPIRE](#)].
- [19] LBNE collaboration, C. Adams et al., *The long-baseline neutrino experiment: exploring fundamental symmetries of the universe*, [arXiv:1307.7335](#) [[INSPIRE](#)].